0. Using quotient rule, $f'(x) = \frac{2(x^2+x-1)}{(2x+1)^2}$ and $f''(x) = \frac{10}{(2x+1)^3}$. Plugging in $x = 0$, we have $f''(x) = 10$
1. This describes half a circle with radius 1 so the arc length would be the circumference or just π. If you didn't recognize that then

$$\int_0^{\frac{\pi}{2}} \sqrt{(\frac{dx}{d\theta})^2 + (\frac{dy}{d\theta})^2}\, d\theta = \int_0^{\frac{\pi}{2}} \sqrt{(-4\cos\theta\sin\theta)^2 + (2\cos^2\theta - 2\sin^2\theta))^2}\, d\theta =$$

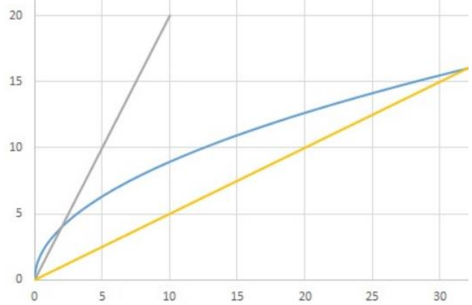
$$2\int_0^{\frac{\pi}{2}} \sqrt{\cos^4\theta + 2\cos^2\theta\sin^2\theta + \sin^4\theta}\, d\theta = 2\int_0^{\frac{\pi}{2}} \sqrt{1}d\theta = \pi$$

2. This is a related rates problem with a right triangle. Let y=the height of the kite, x=the horizontal distance away from Morty, and z=the length of the string of the kite. We have $x^2 + y^2 = z^2$ or $2xdx + 2ydy = 2zdz$. We know that the height of the kite is constant so dy=0, so we have $xdx = zdz$. 60, 91, 109 is a pythagorean triple so x=91. Now, we have (91)(15)=(109)dz, so $dz = \frac{1365}{109}$.
3. Using the disk method, we have $\pi \int_{\frac{1}{2}}^{1} (1 - x^2)\, dx = \frac{5}{24}\pi$
4. Expanding out, we have $\int_{-1}^{1} \frac{y^8 - 4y^7 + 6y^6 - 4y^5 + y^4}{1+y^2} dy = \int_{-1}^{1} y^6 - 4y^5 + 5y^4 - 4y^2 + 4 - \frac{4}{1+y^2} dy = \frac{x^7}{7} - \frac{4x^6}{6} + x^5 - \frac{4x^3}{3} + 4x - 4\tan^{-1}x$ from -1 to 1 $= \frac{160}{21} - 2\pi$
5. Using the nth root test, we have $\lim_{n\to\infty} (\frac{x^{2n+2}}{(3^n+2)(n^4+5)})^{\frac{1}{n}}$. Ignoring the constant values that don't affect the radius, we can simplify down to $\lim_{n\to\infty} \frac{x^2 * x^{1/n}}{3(n^{\frac{1}{n}})(n^{-4})^{\frac{1}{n}}} = \frac{x^2}{3} < 1$

   Thus, the radius of convergence is $\sqrt{3}$.
6. The shape of the building is a prism where a horizontal square slice at a height h has a side length of $30 - 2d = 30 - 2\sqrt{h}$. The distance from the edge of the base is limited to $0 \le d \le 15$ so $0 \le h \le 225$. Now we have a simple integral $\int_0^{225} (30 - 2\sqrt{h})^2 dh$ =33750
7. Factor out an "x" to get $\lim_{x\to\infty} x^2(\left(1 + \frac{7}{x}\right)^{1/7} - \left(1 + \frac{13}{x}\right)^{1/13})$.

   Now use binomial expansion to get $\lim_{x\to\infty} x^2[(1 + \frac{1}{7} * \frac{7}{x} - \frac{3}{49} * (\frac{7}{x})^2 \ldots) - (1 + \frac{1}{13} * \frac{13}{x} - \frac{-6}{169} * (\frac{13}{x})^2 \ldots)$. Every term past the $(\frac{1}{x})^2$ terms will go to zero as x approaches infinity so we can ignore them. Now we can distribute to get $\lim_{x\to\infty} [(x^2 + x - 3) - (x^2 + x - 6) = 3$.

8. The two graphs intersect when $x = 0$ and $x = 4k^3$, so the area is



The area is $\int_0^{4k^3} \sqrt{4kx} - \frac{x}{k}dx = \frac{4\sqrt{k}}{3}x^{\frac{3}{2}} - \frac{x^2}{2k}\Big|_0^{4k^3} = \frac{8}{3}k^5$, which clearly increases throughout the given interval, so we plug in $k = 2$ to get $\frac{256}{3}$.

9. Assume some value

$$A = x^{x^{x^{\cdot^{\cdot^{\cdot^{x^3}}}}}} = 3$$

We can say that

$A = x^A = 3$ and $x^3 = 3$ so $x = 3^{\frac{1}{3}}\ or\ \sqrt[3]{3}$

10. This problem is more easily approached by writing out the first few terms to find a pattern.

$$F(a) = \int_0^1 ((x+1)(x+2)\dots(x+a))\left(\frac{1}{x+1} + \frac{1}{x+2} + \dots + \frac{1}{x+a}\right)$$

is the same as

$\int_0^1 d((x+1)(x+2)\dots(x+a))$ or $(x+1)(x+2)\dots(x+a)$ from $a = 0$ to $a = 1$ which is the same as $(a+1)! - a!$. Plug in $a = 6$ and we get 4320.

11. Expanding out into the three series, the limit simplifies to

$$\lim_{x\to 0}\frac{\left(1+x+\frac{x^2}{2*2!}+\frac{x^3}{2^3*3!}+\cdots\right)-\left(1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots\right)}{1-\left(1-\frac{x^2}{2!}+\frac{x^4}{4!}+\cdots\right)} =$$

$$\lim_{x\to 0}\frac{-\frac{1}{4}x^2-\frac{7}{48}x^3\dots}{\frac{1}{2}x^2-\frac{1}{24}x^4\dots} = \lim_{x\to 0}\frac{-\frac{1}{4}-\frac{7}{48}x\dots}{\frac{1}{2}-\frac{1}{24}x^2\dots} = \frac{-\frac{1}{4}}{\frac{1}{2}} = -\frac{1}{2}$$

12. Using substitution, we have $b = \frac{a^2+a-1}{a^2-a+2}$. Isolating b, $a^2(1-b) + a(1+b) - (1+2b) = 0$. The discriminant of the quadratic must be ≥ 0 so we have $(1+b)^2 - 4\big(-(1+2b)\big)(1-b) = -7b^2 + 6b + 5 \geq 0$. This inequality gets us the interval $[\frac{3-2\sqrt{11}}{7}, \frac{3+2\sqrt{11}}{7}]$. However, f(x) has a maximum value ½ at x=0 so the range is actually $[\frac{3-2\sqrt{11}}{7}, \frac{1}{2}]$.

13. We have $y = \sqrt{x-y}$. $g'^{(20)} = \frac{1}{f'(g(20))} = \frac{1}{f'(420)} = \frac{1}{\frac{1}{41}} = 41$.

14. Substitute $u = 4x^2$ to get $I = \int_0^\infty \frac{\ln(16x^2+1)}{4x^2+1}dx = \frac{1}{4}\int_0^\infty \frac{\ln(4u+1)}{\sqrt{u}(u+1)}dx$

Now, consider the function $F(a) = \int_0^\infty \frac{\ln(au+1)}{\sqrt{u}(u+1)} dx$ where "a" will eventually be 4. We have $F'(a) = \int_0^\infty \frac{\sqrt{u}}{(au+1)(u+1)} du$ with differentiation under the integral sign. Now, integrating we have $F'(a) = \frac{\pi}{a+\sqrt{a}}$ and $F(a) = \int_0^\infty \frac{\pi}{a+\sqrt{a}} da = 2\pi\ln(\sqrt{a}+1)$. We plug back in $a = 4$ so that $F(4) = 2\pi ln3$. Our final answer is $\frac{1}{4}F(4) = \frac{1}{2}\pi ln3$.

0. 10
1. π
2. $\frac{1365}{109}$
3. $\frac{5}{24}\pi$
4. $\frac{160}{21} - 2\pi$
5. $\sqrt{3}$
6. 33750
7. 3
8. $\frac{256}{3}$
9. $\sqrt[3]{3}$ or $3^{\frac{1}{3}}$
10. 4320
11. $-\frac{1}{2}$
12. $[\frac{3-2\sqrt{11}}{7}, \frac{1}{2}]$.
13. 41
14. $\frac{1}{2}\pi ln3$